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SOME MODEL CALCULATIONS OF FRICTIONAL RESISTANCE IN THE MOTION OF BODIES WITH BOUNDARY LAYERS OF VARIABLE VISCOSITY

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Flows when a variable viscosity is present in the boundary layer are of great interest from the applied aspect. In the opinion of a number of authors [1, 2], the motions of marine animals, for which mucus emerges as a substance reducing the viscosity of aqueous solutions, can also serve as analogs of such motions. Some investigations devoted to these questions have been published in [3].

In the present report we give the results of a theoretical investigation of the possible decrease in frictional resistance during flow of the Couette type and during steady and nonsteady flow over a flat plate when at its surface one assigns a concentration of some substance capable of reducing the viscosity of the solution which forms.

1. Let the viscosity vary by the law (Fig. 1)

$$\nu/\nu_0 = \begin{cases} 1 & \text{for } 0 \leqslant |y| \leqslant 1-\alpha, \\ \mathrm{ch}^{-1}\frac{k}{\alpha}(y-1+\alpha) & \text{for } 1-\alpha \leqslant |y| \leqslant 1, \end{cases}$$

where α is the thickness of the diffusional boundary layer; k is some number for which the relative viscosity near the surface is minimal and equal to $\nu/\nu_0|_{y=1} = 1/\cosh k$.

First of all, let us consider flow of the Couette type. In this case

$$\frac{d}{du}\left(\nu\frac{du}{dy}\right) = 0, \quad u(1) = 1, \quad \frac{du(0)}{dy} = 0.$$

The solution has the form

$$u = \frac{1-\alpha+\frac{\alpha}{k}\,\mathrm{sh}\,\frac{k}{\alpha}(y-1+\alpha)}{1-\alpha+\frac{\alpha}{k}\,\mathrm{sh}\,k}.$$

Hence,

$$\tau/\tau_0 = \left(1-\alpha+\frac{\alpha}{k}\,\mathrm{sh}\,k\right)^{-1},$$

where τ_0 is the frictional stress when $\nu = \nu_0$. One can ascertain that $\tau/\tau_0 < 1$ when α, k > 0. However, when $\alpha = 0.1$ and k = 1.7, which corresponds to $\nu/\nu_0|_{y=1} = 0.35$, the relative friction is $\tau/\tau_0 = 0.95$. When $\alpha = 0.1$ and k = 3 ($\nu/\nu_0|_{y=1} = 0.1$), $\tau/\tau_0 = 0.83$; when $\alpha = 0.1$ and k = 3.7 ($\nu/\nu_0|_{y=1} = 0.05$), $\tau/\tau_0 = 0.69$.

Thus, an approximately threefold decrease in the viscosity near the surface is necessary for a significant decrease in resistance which could be noticed experimentally (by 5%).

A similar result is obtained in an analysis of Poiseuille flow.

2. Let us consider the case of nonsteady motion. Imagine an infinite plate suddenly set into motion.

The equation of motion and the equation for the diffusion in the boundary layer have the form

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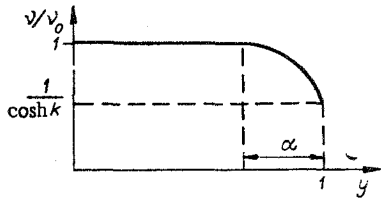

Fig. 1

TABLE 1

ε \ $\mathrm{Pr_d}$	0,1	1,0
0,3	0,85	0,88
0,9	0,325	0,36

$$\frac{d}{dy}\left[\nu(c)\frac{\partial u}{\partial y}\right]=\frac{\partial u}{\partial t},\quad \chi\frac{\partial^2 c}{\partial y^2}=\frac{\partial c}{\partial t},\tag{2.1}$$

where χ is the coefficient of diffusion; c is the concentration of the substance in the boundary layer. We assume that the form of the dependence $\nu(c)$ of the viscosity on the concentration in Eq. (2.1) is given.

The boundary conditions are

$$u=c=0 \text{ for } t<0,\ 0<y\leqslant\infty,$$
$$u=0,\ c=c_0 \text{ for } t=0,\ y=0,$$
$$u=U_0,\ c=c_0 \text{ for } y=0,\ 0<t\leqslant\infty.$$

We seek the solution in the form

$$u=U_0u(\eta),\ c=c_0c(\eta),\ \eta=y/2\sqrt{\nu_0 t}.$$

We obtain the system of equations

$$[\nu(c)u']'+2\eta u'=0,\ c''+2\,\mathrm{Pr}_d\eta c'=0,$$

where $\nu(c)$ is the viscosity normalized to ν_0; $\mathrm{Pr_d}=\nu_0/\chi$ is the diffusional Prandtl number.

The solution has the form

$$u=1-\int\limits_0^\eta \nu(c)\exp\left[-2\int\limits_0^t \nu^{-1}(c)\beta d\beta\right]dt\quad\left\{\int\limits_0^\infty \nu^{-1}(c)\exp\left[-2\int\limits_0^t\nu^{-1}(c)\beta d\beta\right]dt\right\}^{-1},\tag{2.2}$$

$$c=1-\frac{2}{\sqrt{\pi \mathrm{Pr}_d}}\int\limits_0^{\sqrt{\mathrm{Pr}_d}\eta}\exp(-t^2)\,dt,$$

from which the ratio of shear stresses is expressed in the form

$$\tau/\tau_0=\frac{\sqrt{\pi}}{2}\left\{\int\limits_0^\infty \nu^{-1}(c)\exp\left[-2\int\limits_0^t\nu^{-1}(c)\beta d\beta\right]dt\right\}^{-1}.\tag{2.3}$$

If $\nu(c)$ is assigned in the form

$$\nu(c)=1-2\varepsilon c+\varepsilon c^{2}*$$

(in this case the minimum viscosity is obtained for $c=1$, i.e., at the boundary, and equals $(1-\varepsilon)$), then a calculation leads to the values of τ/τ_0 given in Table 1. In the calculation we did not use Eq. (2.3) but numerically integrated (2.2) and found the quantity $\nu\,\partial u/\partial y\,|_{y=0}$.

Thus, in this case also a decrease in viscosity by at least 30% is necessary for a decrease in friction by 10-15%.

*We note that additions usually increase the viscosity of solutions. There are substances, however, whose aqueous solutions have a lower viscosity than that of water. An example of such substances is the mucus at the surfaces of fish [3, 4]. The influence of mucus has a significant place among the existing hypotheses about the mechanism of a possible decrease in the resistance to the motion of aquatic animals.

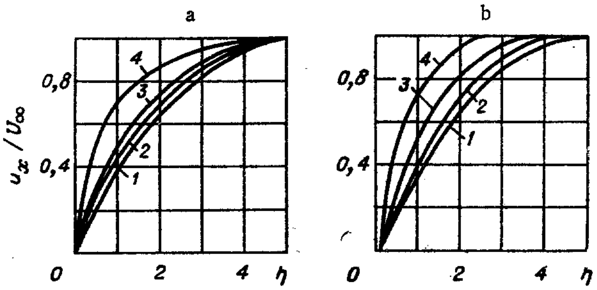

Fig. 2

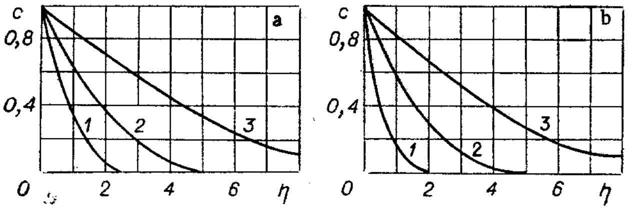

Fig. 3

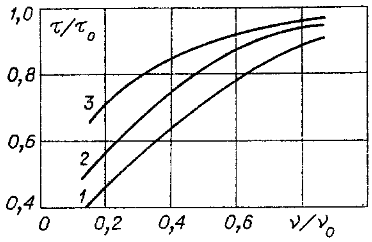

Fig. 4

3. Let us consider an analog of the Blasius problem of flow over a semiinfinite plate for the case when a concentration of some substance capable of decreasing the viscosity of the solution which forms is assigned at the surface of the plate. Assuming that the decrease in viscosity is proportional to the change in concentration and to the velocity, we find that the viscosity varies by the exponential law

$$\nu = \nu_0 e^{-\varepsilon c}, \tag{3.1}$$

where c is the concentration of the substance; ν_0 is the viscosity of the undisturbed stream.

Introducing the Blasius variable $\eta = y\sqrt{U_\infty/\nu_0 x}$ and the dimensionless velocity components

$$u = \frac{u_x}{U_\infty} = f'(\eta), \quad v = \frac{u_y}{U_\infty} = \frac{1}{2}\sqrt{\frac{\nu_0 U_\infty}{x}}(\eta f' - f),$$

we reduce the equations of motion and diffusion to the system of ordinary differential equations

$$\{\nu(c) f''\}' + \frac{1}{2} f f'' = 0, \quad c'' + \frac{1}{2}\mathrm{Pr}_d f c' = 0 \tag{3.2}$$

with the boundary conditions

$$\begin{aligned} f = f' = 0,\ c = 1 \ \text{for}\ \eta = 0; \\ f' = 1,\ c = 0 \ \text{for}\ \eta = \infty. \end{aligned} \tag{3.3}$$

The system of equations (3.2) with the boundary conditions (3.3) and the relation (3.1) was integrated numerically on a computer for different values of the Prandtl number $\mathrm{Pr_d}$ and the constant ε.

Some characteristic velocity profiles are shown in Fig. 2: a) $\mathrm{Pr_d}$ = 10: 1) α = 0.327, ε = 0.1; 2) α = 0.307, ε = 0.5; 3) α = 0.277, ε = 1.0; 4) α = 0.206; ε = 2.0; b) $\mathrm{Pr_d}$ = 0.1: 1) α = 0.31, ε = 0.1; 2) α = 0.262, ε = 0.5; 3) α = 0.21, ε = 1.0; 4) α = 0.13, ε = 2.0.

The variation in the concentration is shown in Fig. 3: a) ε = 0.1; 1) $\mathrm{Pr_d}$ = 10, α = 0.327; 2) $\mathrm{Pr_d}$ = 1.0,

α = 0.323; 3) Pr_d = 0.1, α = 0.31; b) ε = 2.0: 1) Pr_d = 10, α = 0.206; 2) Pr_d = 1.0, α = 0.157; 3) Pr_d = 0.1, α = 0.126.

We note that when ε = 0.1 and Pr_d = 10 and 0.1 the corresponding velocity profiles differ from Blasius profiles by no more than 2%. The most interesting is the relative friction τ/τ_0 at the plate surface, where τ_0 is the friction for ordinary Blasius flow. The quantity τ/τ_0 is shown in Fig. 4 (curves 1-3 correspond to Pr_d = 0.1, 1.0, and 10), from which it is seen that an order-of-magnitude decrease in viscosity is required for an appreciable decrease in resistance, such as by 50%.

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DIAGNOSIS OF THE FUNDAMENTAL TURBULENT CHARACTERISTICS OF TWO-PHASE FLOWS

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In the experimental investigation of two-phase gas–liquid flows there is today a noticeable shift from the measurement of averaged characteristics (pressure drop, average gas content, and average heat-transfer coefficient) to the detailed study of the turbulent structure of the flow. What is of interest is to determine the local values of gas content, the phase velocities, the frictional stresses at the wall, and pulsation and spectral characteristics.

Among the most detailed investigations in this field we should include [1-3], published during the past few years, which give the results of measurements of local gas content and liquid and gas velocities, as well as the intensity of the velocity pulsations. The work is being done mainly by means of a thermoanemometer, by the electrical-conductivity method and partly by means of optical probes.

For a number of years the Thermophysics Institute of the Siberian Branch of the Academy of Sciences of the USSR has been conducting detailed investigations of the turbulent characteristics of gas–liquid flows. The methods used are based on electrochemical diagnosis, which makes it possible to carry out measurements of the average values and pulsations of the tangential stress at the wall and also to increase the resolving power in the measurement of phase velocities. In this way it is possible to supplement to a considerable degree the information obtained by the methods mentioned above and to obtain a more detailed picture of the flow.

In the present study we describe a method for determining the main characteristics of a gas–liquid flow.

1. Tangential Stress at the Wall. The electrochemical method of determining the tangential stress at the wall [4-6] consists in the following. Two electrodes – a small cathode and an anode – are placed in a stream of electrolyte of a special composition. The cathode serves as a sensor for determining the tangential stress and consists of a small segment of platinum or nickel wire or plate embedded flush with the wall of the channel. The application of voltage to the electrodes starts a rapid electrochemical reaction, which results in polarization of the cathode. For the case of the most widely used electrolyte composition, which consists of a

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